The distribution of values of logarithmic derivatives of real L-functions

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Abstract

In this thesis we prove three main results. The first result is on omega theorems for $L'/L(1,\chi_D)$, where D is a fundamental discriminant, and χ_D is the real character attached to D. These theorems describe large values of L'/L, and their influence on the behaviour of the Euler-Kronecker constant. We prove unconditionally that

$$\frac{L'}{L}(1,\chi_D) \geq \log\log|D| + O(1).$$

holds for infinitely many fundamental discriminants D. Furthermore, we prove that for a positive proportion of fundamental discriminants D we have

$$\frac{L'}{L}(1,\chi_D) \leq -\log\log D + O(1).$$

We also prove under GRH, that for infinitely many primes $q \equiv 1 \mod 4$ we have

$$\frac{L'}{L}(1,\chi_q) \geq \log\log q + \log\log\log q + O(1)$$

and for infinitely many primes $q \equiv 1 \mod 4$ we have

$$\frac{L'}{L}(1,\chi_q) \leq -\log\log q - \log\log\log q + O(1).$$

In the second result, we prove that the moments of L'/L are constant. In other words, for each non-negative integer k, there is a constant C_k so that

$$\sum_{0<\beta D\leq Y}^{*} \left(-\frac{L'}{L}(1,\chi_D)\right)^k \sim C_k Y$$

The third result which is on the distribution of values of $L'/L(\sigma,\chi_D)$ shows that there is a density function $\mathcal{Q}_\sigma$ such that for $\sigma > \frac{1}{2}$

$$\#\{D \text{ fundamental discriminants, such that } |D| \leq Y, \text{ and } \alpha \leq L'/L(\sigma,\chi_D) \leq \beta\}$$

$$\sim \frac{3}{\pi^3} Y \int_\alpha^\beta \mathcal{Q}_\sigma(x)dx.$$